Convex Optimization

Lab 6: Convex Functions

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Autumn Semester 2024

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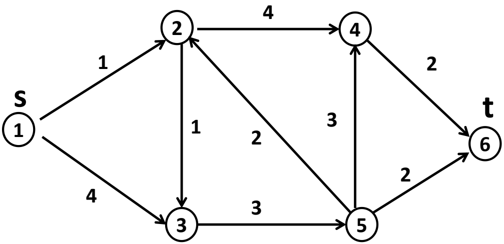
Outline

1. Solve the Network Max-flow Problem
2. Convex Set
3. Convex functions

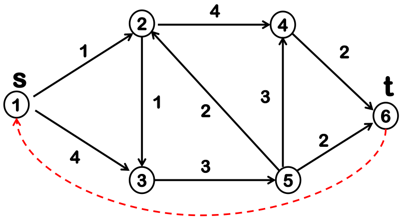
Network Max-flow Problem: the motivation

- Given an oil pipeline network as follows
- What is the maximum sent, per hour, from source to a sink



- Given a directed, capacitated network $G = (N, A)$ with arc capacities $u_{ij} \geq 0, \forall (i, j) \in A$, determine the maximum possible amount of flow from a designated source node **s** to a sink node **t** while obeying all arc capacities

Network Max-flow Problem: the model



var	index	var	index
x_{12}:	1	x_{24}:	6
x_{13}:	2	x_{54}:	7
x_{23}:	3	x_{56}:	8
x_{35}:	4	x_{46}:	9
x_{52}:	5		

$$\text{Max. } x_{46} + x_{56}$$

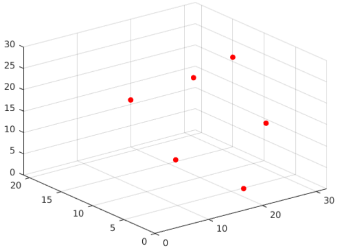
$$\text{s.t.} \begin{cases} x_{12} + x_{52} - x_{24} - x_{23} = 0 \\ x_{13} + x_{23} - x_{35} = 0 \\ x_{24} + x_{54} - x_{46} = 0 \\ x_{35} - x_{52} - x_{54} - x_{56} = 0 \\ x_{ij} \leq u_{ij} \\ x_{ij} \geq 0 \end{cases} \tag{1}$$

Display the Convex Hull of following 3D points (1)

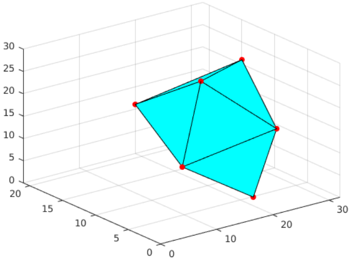
- [5 1 15;23 2 17;32 15 21;20 21 11;13 5 29;20 3 2]
- Display a group of 3D points and their convex hull

```
function conhull()
    clf;
    P = [5 1 15;23 2 17;32 15 21;20 21 11;13 5 29;20 3
   2];
    [k, vol] = convhulln(P);
    trisurf(k,P(:,1),P(:,2),P(:,3),'FaceColor','cyan')
end
```

Display the Convex Hull of following 3D points (2)



(a)



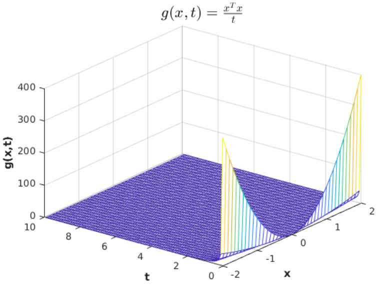
(b)

Draw following functions by Matlab

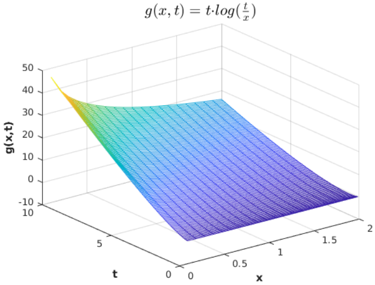
1. Exponential function: $f(x) = e^{2x}$, $x \in [-5, 5]$
2. Power function: $f(x) = x^{1.1}$, $x \in (0, 5]$
3. Power function: $f(x) = x^{0.5}$, $x \in (0, 5]$
4. Absolute power: $f(x) = |x|^{1.5}$, $x \in [-5, 5]$
5. Logarithmic: $f(x) = ln(x)$, $x \in (0, 50]$
6. Negative entropy: $f(x) = xlog_2(x)$, $x \in (0, 10]$

Draw Perspective Projection functions by Matlab

1. Perspective Projection: $g(x,t) = \frac{x^T x}{t}$, $x \in [-2, 2]$, $t(0, 10]$
2. Perspective Projection: $g(x,t) = t \cdot log(\frac{t}{x})$, $x \in (0, 2]$, $t(0, 10]$



(c)



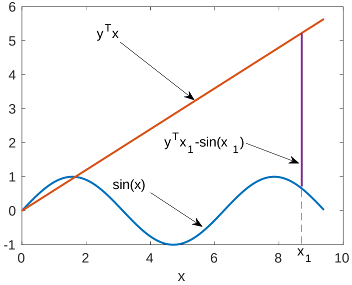
(d)

Draw Conjugate functions by Matlab (1)

- Given function $f(x)$, $R^n \rightarrow R$, the conjugate function is defined as

$$f^*(y) = sup_x(y^T x - f(x)), R^n \rightarrow R \quad (2)$$

1. Given $f(x) = x^2$
2. Given $f(x) = xln(x)$



Draw Conjugate functions by Matlab (2)

- Given function $f(x), R^n \rightarrow R$, the conjugate function is defined as

$$f^*(y) = sup_x(y^T x - f(x)), R^n \rightarrow R \tag{3}$$

1. Given $f(x) = x^2 \quad \longrightarrow \quad f^*(y) = y{\cdot}x - x^2$
2. Given $f(x) = xln(x) \quad \longrightarrow \quad f^*(y) = y{\cdot}x - xln(x)$

Now, let's try to find out the superum of the functions w.r.t x

$$\Downarrow$$

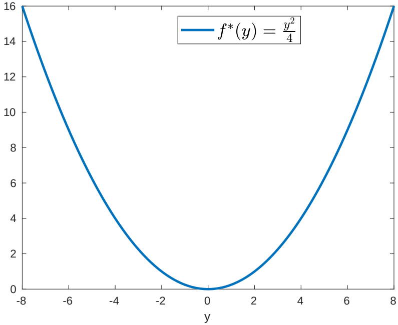
3. $\frac{\partial f^*(y)}{\partial x} = y - 2x = 0 \quad \longrightarrow \quad x = \frac{y}{2}$
4. $\frac{\partial f^*(y)}{\partial x} = y - 1 - ln(x) = 0 \quad \longrightarrow \quad x = e^{y-1}$

$$\Downarrow$$

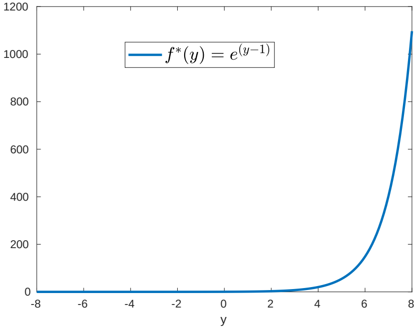
5. $f^*(y) = \frac{y^2}{4}$
6. $f^*(y) = e^{y-1}$

Draw Conjugate functions by Matlab (3)

5. $f^*(y) = \frac{y^2}{4}$
6. $f^*(y) = e^{y-1}$



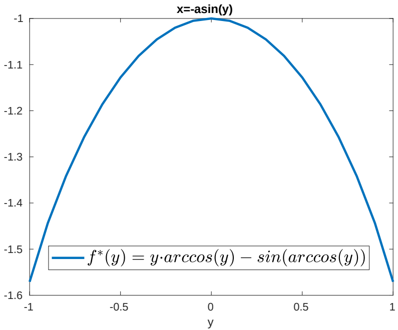
(e)



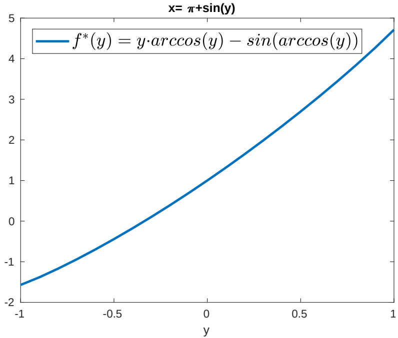
(f)

Draw Conjugate functions by Matlab (4)

- Given $f(x) = cos(x)$, plot out $f^*(y) = sup_x(y \cdot x - cos(x))$
- Given $f(x) = sin(x)$, plot out $f^*(y) = sup_x(y \cdot x - sin(x))$
- Given $f(x) = e^{-\frac{x^2}{2}}$, plot out $f^*(y) = sup_x(y \cdot x - e^{-\frac{x^2}{2}})$



(g)



(h)