Convex Optimization

Lab 5: Linear Programming (3)
Two-phase Simplex

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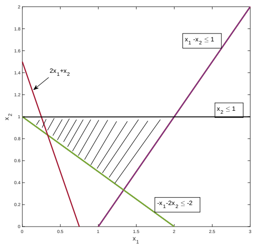
Outline

Linear Programming: the two-phase problem (1)

$$\text{Max. } 2x_1 + x_2$$

$$\text{subject to} \begin{cases} x_1 - x_2 \leq 1 \\ -x_1 - 2x_2 \leq -2 \\ x_2 \leq 1 \\ x_1, x_2 \geq 0 \end{cases} \quad (1)$$



	z	x_1	x_2	s_1	s_2	s_3	
	1	-2	-1	0	0	0	0
s_1	0	1	-1	1	0	0	1
s_2	0	-1	-2	0	1	0	-2
s_3	0	0	1	0	0	1	1

- $x_1 = x_2 = 0$ is not a basic solution for the problem
- There is no start point

Two-Phase Tableau: Problem-1

- Solve following problem by Two-Phase Tableau
- Please solve it with Spreadsheet
- Verfiy your answer with Matlab 'linprog'

$$\text{Max. } 3x_1 - x_2 + 2x_3$$
$$\text{s. t.} \begin{cases} x_1 + 3x_2 \leq 5 \\ 2x_1 - x_2 + x_3 \geq 2 \\ 4x_1 + 3x_2 - 2x_3 = 5 \\ x_1, x_2, x_3 \geq 0 \end{cases} \tag{2}$$

Answer

```
function x = prob1()
    f = -1*[3 -1 2];
    A = [1 3 1;-2 1 -1];
    b = [5 -2]';
    Ae = [4 3 -2];
    be = [5];
    x = linprog(f, A, b, Ae, be, zeros(2,1), []);
    -1*f*x
end
```

Two-Phase Tableau: Problem-2

- Solve following problem by Two-Phase Tableau
- Please solve it with Spreadsheet
- Verfiy your answer with Matlab 'linprog'

$$\text{Max. } 2x_1 - 6x_2$$
$$\text{s. t.} \begin{cases} -x_1 - x_2 - x_3 \leq -2 \\ 2x_1 - x_2 + x_3 \leq 1 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

$$\text{Max. } 2x_1 - 6x_2$$
$$\text{s. t.} \begin{cases} -x_1 - x_2 - x_3 + s_1 - a_1 = -2 \\ 2x_1 - x_2 + x_3 + s_2 = 1 \\ x_1, x_2, x_3, s_1, s_2, a_1 \geq 0 \end{cases}$$

Answer

```
function x = prob2()
    f = -1*[2 -6 0];
    A = [-1 -1 -1;2 -1 1];
    b = [-2 1]';
    Ae = [];
    be = [];
    x = linprog(f, A, b, Ae, be, zeros(3,1), []);
    -1*f*x
end
```

Two-Phase Tableau: Problem-3

- Solve following problem by Two-Phase Tableau
- Please solve it with Spreadsheet
- Verfiy your answer with Matlab 'linprog'

$$\text{Max. } 2x_1 + 3x_2$$
$$\text{s. t. } \begin{cases} -4x_1 + 3x_2 \leq 12 \\ 2x_1 + x_2 \leq 6 \\ x_1 + x_2 \geq 3 \\ 5x_1 + x_2 \geq 4 \\ x_1, x_2 \geq 0 \end{cases}$$

$$\text{Max. } 2x_1 + 3x_2$$
$$\text{s. t. } \begin{cases} -4x_1 + 3x_2 + s_1 \leq 12 \\ 2x_1 + x_2 + s_2 \leq 6 \\ -x_1 - x_2 + s_3 - a_1 \leq -3 \\ -5x_1 - x_2 + s_4 - a_2 \leq -4 \\ x_1, x_2, s_1, s_2, s_3, s_4, a_1, a_2 \geq 0 \end{cases}$$

General steps in Two-phase Simplex

A. Stage-1: Solve the auxiliary problem
   1. Set k=1, Repeat
   2. Find out r by argmin$_r\{B^{-1}\cdot b./B^{-1}\cdot N(:,k)\}$
   3. Swap-in $N(:,k)$ to B, Swap-out $B(:,r)$ to N_2
   4. $Cn(k) \rightleftharpoons Cb(r)$
   5. Find out k by argmin$_k\{C_b\cdot B^{-1}N - C_n\}$

B. State-2: Solve the original problem
   1. Set $C_b = [0 \ \cdots \ 0]$, $C_n = -C\cdot N\cdot B^{-1}$
   2. $B_2 = N, N_2 = B^{-1}$, $b = B^{-1}\cdot b$
   3. Repeat
   4. Find out k by argmin$_k\{C_b\cdot B_2^{-1}N_2 - C_n\}$
   5. Find out r by argmin$_r\{B_2^{-1}\cdot b./B_2^{-1}\cdot N_2(:,k)\}$
   6. Swap-in $N_2(:,k)$ to B_2, Swap-out $B_2(:,r)$ to N_2
   7. $Cn(k) \rightleftharpoons Cb(r)$

Linear Programming: implement Two-phase Simplex (2)

1. Implement function phase1(), return B, N, b
2. Implement function phase2(), return B, N, b

```
function [fval, Xb]=simplexP2(A, b, C)
    %design a loop to run the Simplex procedure
    %define matrix B
    %define matrix N
    %define Cb, Cn
    %k=1
    %[B, N, b] = phase1(B, Cb, N, Cn, b);
    %Cb = [0 0 0];
    %Cn=-C*N*inv(B);
    %[B, N, b] = phase2(B, Cb, N, Cn, b);
    %fval = C*b;
end
```

Linear Programming: implement Two-phase Simplex (3)

1. Implement function phase1(), return B, N, b
2. Implement function phase2(), return B, N, b

```
function [B, N, b] = phase1(B, Cb, N, Cn, b)
    %k=1
end

function [B, N, b] = phase2(B, Cb, N, Cn, b)
    %implement by yourself
end
```