Convex Optimization

Lab 4: Linear Programming (2)
Simplex Method

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Outline

1. Solve LP by Simplex Method

Linear Programming: Simplex Method (1)

- Given $C_b = [0 \quad 0 \quad 0], C_n = [7 \quad 4]$

$$
B = \begin{bmatrix} x_3 & x_4 & x_5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{m \times m} \qquad N = \begin{bmatrix} x_1 & x_2 \\ 2 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}_{m \times p} \qquad b = \begin{bmatrix} 20 \\ 18 \\ 8 \end{bmatrix}_{m \times 1}
$$

1. Find out $k = \text{argmin}_{j \in R} \{ C_b \cdot B^{-1} \cdot p_j - c_j \}$
2. $\overline{b} = B^{-1} b$, $y_k = B^{-1} p_k$
3. Find out $r = \text{argmin}_i \{ \frac{\overline{b}_1}{y_{1k}}, \cdots, \frac{\overline{b}_i}{y_{ik}}, \cdots, \frac{\overline{b}_m}{y_{mk}} \}$
4. $B(: r) \rightleftharpoons N(:, k)$
5. Swap corresponding columns in C_b and C_n

Task: implement Simplex (1)

- Given $C_b = [0 \quad 0 \quad 0], C_n = [7 \quad 4]$

$$B = \begin{bmatrix} x_3 & x_4 & x_5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{m \times m} \qquad N = \begin{bmatrix} x_1 & x_2 \\ 2 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}_{m \times p} \qquad b = \begin{bmatrix} 20 \\ 18 \\ 8 \end{bmatrix}_{m \times 1}$$

```matlab
function [fval, Xb]=simplex(A, b, C)
    %design a loop to run the Simplex procedure
end
```

Linear Programming: Task: implement Simplex (2)

❶ Find out $k = \mathrm{argmin}_{j \in R}\{C_b \cdot B^{-1} \cdot p_j - c_j\}$

❷ $\overline{b} = B^{-1}b$, $y_k = B^{-1}p_k$

❸ Find out $r = \mathrm{argmin}_i\{\frac{\overline{b_1}}{y_{1k}}, \cdots, \frac{\overline{b_i}}{y_{ik}}, \cdots, \frac{\overline{b_m}}{y_{mk}}\}$

❹ $B(:r) \rightleftharpoons N(:,k)$

❺ Swap corresponding columns in C_b and C_n

```
1  function [fval, Xb]=simplex(A, b, C)
2      %design a loop to run the Simplex procedure
3      %define matrix B
4      %define matrix N
5      %define Cb, Cn
6      % Repeat
7      %    1. Find out xk
8      %    2. Find out r
9      %    3. Swap-in xk and Swap-out X_Br
10     %    4. Swap ck into Cb, swap-out Cr to Cn
11 end
```