Assignment-4

1. A set C is midpoint convex if whenever two points a, b are in C, the average or midpoint (a + b)/2 is in C. Obviously a convex set is midpoint convex. It can be proved that under mild conditions midpoint convexity implies convexity. As a simple case, prove that if C is closed and midpoint convex, then C is convex.

2. Let $C \subseteq R^n$ be a convex, with $x_1, \cdots, x_k \in C$, and let $\theta_1, \cdots, \theta_k \in R$ satisfy $\theta_i \geq 0$, $\theta_1 + \cdots + \theta_k = 1$. Show that $\theta_1 x_1 + \cdots + \theta_k x_k \in C$ for $k \geq 3$.

3. Show that the convex hull of a set S is the intersection of all convex sets that contain S.

4. Given function $f(x) = -5x^2 + 5x$, work out its conjugate function and plot it out

5. Suppose $f : S \subset R \to R$ is convex. Let $a,\ b \in S$ and $a < b$. Show

$$f(x) \leq \frac{b-x}{b-a} f(a) + \frac{x-a}{b-a} f(b), \quad \forall x \in [a, b] \tag{1}$$

- Hints

  1. Submission due: 2024/Nov./30
  2. Submit to lecwlzhao@163.com, email title "assigment4_your-name + your student number"